Angular Momentum and the Principle of Relativity: Principle of Relativity Consistency vs. Lorentz Violation

Vesselin C. Noninski

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Abstract

The angular momentum of a particle orbiting circularly in inertial frame K is examined, finding $L_z = mr^2\omega$ consistently transforms to $L'_z = mr^2\omega$ in frame k under Galileo's framework, upholding the foundational principle of relativity (PoR) for any frame velocity $\boldsymbol{v}$. Applying Lorentz transformations (LT) to transfer L'_z back to K, however, yields a distinct L_z, creating an impossible duality in K at one time. This violation of PoR by LT challenges the coherence of relativistic angular momentum, contrasting sharply with PoR invariance.

I. PoR and Angular Momentum Consistency

Consider a particle of mass m in frame K, orbiting circularly in the xy-plane around the origin:

$$\mathbf{r} = (r\cos(\omega t), r\sin(\omega t), 0), \quad \mathbf{u} = (-r\omega\sin(\omega t), r\omega\cos(\omega t), 0),$$

$$\mathbf{p} = m\mathbf{u}, \quad \mathbf{L} = \mathbf{r} \times \mathbf{p} = (0,0,mr^2\omega).$$

In frame k (primed frame), moving at $\mathbf{v} = (v_x, v_y, v_z)$ relative to K, PoR gives:

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}\mathrm{t}, \quad \mathbf{u}' = \mathbf{u} - \mathbf{v}.$$

Relative to the orbit's center (at $-\mathbf{v}t$ in k):

$$\mathbf{r}'_{\mathbf{rel}} = \mathbf{r}' - (-\mathbf{v}t) = \mathbf{r}, \qquad \mathbf{u}'_{\mathbf{rel}} = \mathbf{u}' - (-\mathbf{v}) = (\mathbf{u} - \mathbf{v}) + \mathbf{v} = \mathbf{u},$$

$$\boldsymbol{L}' = \boldsymbol{r}'_{\boldsymbol{rel}} \times m\boldsymbol{u}'_{\boldsymbol{rel}} = (0,0,mr^2\omega)$$

Initially, we erred by computing $\mathbf{L}' = \mathbf{r}' \times m\mathbf{u}'$, yielding $\mathbf{L} + m(\mathbf{r} \times \mathbf{v}) - mt(\mathbf{v} \times \mathbf{u})$, suggesting a violation of PoR. This mistake stemmed from using k's absolute origin (the origin of K) rather than the orbit's center in k, misaligning with K's setup, producing a cycloidal path instead of a circular path.

In calculating what the PoR-transformed expression for the angular momentum in k should be, we must keep the following in mind. The radius vector consists of a beginning, which is the origin of the frame, and an endpoint. The radius vector is expressed by a difference: (end-point) – (origin of the frame). In K, where the origin around which the circular motion takes place is (0,0,0), the radius-vector is $(\mathbf{r} - (0,0,0))$ (or the endpoint

$(r\cos(\omega t), r\sin(\omega t), 0)$ minus the origin (0,0,0)). Now, if we want to express the radius vector in k in terms of K, we must correct first the endpoint of the radius vector in k in terms of K by always subtracting distance $(\mathbf{v}\, t)$ from $\mathbf{r}$ (that is, for the endpoint of the radius vector we must write $(\mathbf{r} - \mathbf{v}\, t)$, which in our case is: $(x' = r\cos(\omega t) - v_x t, \quad y' = r\sin(\omega t) - v_y t, \quad z' = 0)$ because the motion is in the xy-plane and the component along the z-axis is zero), but we must also correct the origin of k if we want to express this origin in terms of K, that is, we must subtract the distance traveled by k in K, $(\mathbf{v}t)$, making the origin of k in terms of K to be $(-\mathbf{v}t)$, which is the origin of k expressed in terms of K. So, finally, the radius-vector in k in terms of K is $(\mathbf{r} - \mathbf{v}\, t) - (-\mathbf{v}t) = \mathbf{r}$, which results in $(r\cos(\omega t), r\sin(\omega t), 0)$. The important conclusion is that in Galileo' framework, which is proper, the velocity $\mathbf{v}$ is immaterial in transformations across inertial frames. This result dispels the impression that the laws of physics are not invariant across inertial frames under transformation in Galileo's framework (GF).

Thus, correctly, $L_z = L'_z = mr^2\omega$ across frames, as $\mathbf{v}$ cancels when referencing the physical orbit, upholding PoR: since $\boldsymbol{u}' = \boldsymbol{u} - \boldsymbol{v} + \boldsymbol{v} = \boldsymbol{u}$ relative to the center of rotation, which is the origin of k, expressed in terms of K, the law $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ applies invariantly, and the value remains consistent.

II. Lorentz Transformations: A Violation of PoR

In K, $L_z = \mathrm{m}r^2\omega$ (z-axis component, with x- and y-components zero) is also $L'_z = mr^2\omega$ in k (z'-axis component, x'- and y'-components zero). The PoR used for this transformation, foundational to relativity, ensures physical laws' uniformity. We now apply the inverse LT to transfer this back to K ($\mathbf{v} = (v, 0,0)$):

$$x = \gamma(x' + vt'), \quad y = y', \quad t = \gamma\left(t' + \frac{vx'}{c^2}\right), \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$u_x = \frac{u'_x + v}{1 + \frac{vu'_x}{c^2}}, \quad u_y = \frac{u'_y}{\gamma\left(1 + \frac{vu'_x}{c^2}\right)},$$

respectively,

$$\mathbf{r}'_{rel} = (r\cos(\omega t'),\ r\sin(\omega t'),\ 0), \qquad \mathbf{u}'_{rel} = (-r\omega\sin(\omega t'),\ r\omega\cos(\omega t'),\ 0),$$

with the aim to transfer this $L'_z = mr^2\omega$ in k back to K, obtaining the fixed value:

$$L_z = \gamma_u \mathrm{m}r^2\omega, \text{ where } \gamma_u = \frac{1}{\sqrt{1 - \frac{(r\omega)^2}{c^2}}}$$

This produces two distinct L_z in K at the same time t:

1. $L_z = \gamma_u m r^2 \omega$,

2. $L_z = m r^2 \omega$

This is impossible—one particle in one frame (here frame K), at one and the same time cannot have two distinct angular momenta. PoR, upheld by $L_z = L'_z = m r^2 \omega$ in Galileo's framework, is violated by LT, which introduce γ_u and v-dependence absent in the foundational equality. Noninski [1] critiques this as a flaw in LT, fracturing the unity PoR demands, unlike PoR consistency.

Pondering Non-Relativistic Solution

Upon considering [1,2] and the current study one may see the true puzzle's depth.

Resolution of the three puzzles in [2] and any other puzzles, to say nothing of the foundational discrepancy found in [1], and here can come about only by radically abandoning LT-based theories as the most important step.

When pondering a non-relativistic approach to momentum flux and generation of momentum, now that theories governed by LT are shown untenable, one, indeed, may be tempted to go along the trodden path of proposing as the initial point an inhomogeneous, Poisson-like, wave equations such as $\nabla^2 \psi - \frac{1}{v_g^2}\frac{\partial^2 \psi}{\partial t^2} = -4\pi G \rho$. It has a gravitational constant G and mass density ρ source term $-4\pi G \rho$. The latter is made equal to a Laplacian ∇^2 of a gravitational scalar potential field ψ (alluding to how mass generates gravitational field: $\nabla^2 \psi = 4\pi G \rho$) with time-dynamics of ψ, $\frac{\partial^2 \psi}{\partial t^2}$, a wave-like propagation multiplied by the inverse square of the wave speed v_g (not c), suggesting a hypothetical graviton-like scalar field with speed v_g, subtracted from it. Then, it may be suggested that the gravitational perturbation field ψ with propagation speed v_g is driven by a quadrupole moment tensor $Q_{ij} = \mu\left(x^i x^j - \frac{1}{3}\delta_{ij} r^2\right)$ with $Q_{xx} = \mu r^2 \cos(2\omega t)$, $\quad Q_{xy} = \mu r^2 \sin(2\omega t)$ … defining circular orbit with radius r and the flux $\frac{dL_z}{dt} = \int_S \left(\boldsymbol{r} \times \boldsymbol{p}_{field}\right) \cdot d\boldsymbol{A} \rightarrow \frac{dL_z}{dt} = -\frac{32}{5}\frac{G^{5/2}\mu^2 M^{3/2}}{v_g r^{7/2}}$, where the flux momentum is $\boldsymbol{p}_{\mathbf{field}} \propto -\frac{1}{v_g}\dot{\psi}\nabla\psi$, achieving resemblance with GR's $-\frac{32G^2\mu^2 M}{5c^5 r^3}$, with v_g replacing c^5's role. This must necessarily be accompanied by unpacking the hermetic doubtful beauty of the cute reductions of math such as tensors—the truth is beautiful, shells concealing wrongness are not. As we saw the devil is in details cloaked by the mathematical constructs. This should be a call for a return to physics, no matter how inauspicious and far from advanced it may sound, and leave mathematics as only the helping hand. Mathematics cannot create reality, it can only describe reality when the latter is established through the methods of physics.

As an illustration, besides the shown problem, even contemporary non-relativistic models use tensors (e.g., $T_{r\phi}$) for mass and momentum flux over a surface: $\frac{dL_z}{dt} = \int_S r\sin\theta\, T_{r\phi} R^2 \sin\theta\, d\theta d\phi$, but tensors assume $F = ma$, a static balance (third law), not motion.

The true motion requires:

$$\mathrm{F}_{\mathrm{real}} = \mathrm{ma} + \frac{mv^2}{2x},$$

from definitions: $v = \frac{dx}{dt}$, $a = \frac{dv}{dt}$, for constant a:

$$\int v\,dv = \int a\,dx \;\rightarrow v^2 = 2ax \rightarrow \underbrace{\mathrm{m}v^2}_{v^2 \rightarrow c^2} = \underbrace{\underbrace{2ma}_{2F=F_{\mathrm{real}}}\;x}_{\text{Energy, } E} \rightarrow \mathrm{F}_{\mathrm{real}} = \mathrm{ma} + \frac{mv^2}{2x}$$

revealing $E = mc^2$ (with c as a limit velocity) as classical, not relativistic. By extension, the above omission propagates into the Lagrangians, Hamiltonians, and the rest of the machinery used in describing nature. Tensors hide this, and LT compound it with frame inconsistencies. These flaws suggest Javadinezhad and Porrati's tensor-based flux [2] may inherit similar issues.

Conclusion

The framework of PoR preserves $L_z = mr^2\omega$ across frames. LT disrupt this, yielding dual L_z in K, an absurd outcome challenging relativistic angular momentum's validity in contexts like [2].

The findings here do indeed point to a deep crack in the relativistic framework. It's not just a technical glitch; it challenges the coherence of relativity at its root, suggesting that any researcher taking this seriously might indeed hesitate to engage with relativity further.

The observed duality—two distinct L_z for one particle in one frame at the same time—is not just odd; it's logically impossible. PoR, foundational to both non-relativistic and relativistic physics, insists on a single, frame-consistent value. LT's introduction of γ_u, tied to the frame velocity $\mathbf{v}$, fractures this unity, violating the very principle relativity claims to uphold. In PoR terms, $\mathbf{v}$ is immaterial to the intrinsic orbital motion; in LT, $\mathbf{v}$ alters the outcome, suggesting relativity imposes an artificial frame dependence that undermines its own logic—an internal contradiction no experiment can salvage. This isn't a minor puzzle—it's a collapse of the framework's ability to describe physical reality sensibly. A sensible researcher, seeing this, might indeed question why they'd invest in a theory that can't resolve such a basic quantity without contradiction. If an internal contradiction is accepted as true then anything can be true, erasing the distinction between truth and falsity.

The closest in [2] to the findings here is the discovered mismatch between transformed and intrinsic values in one frame. It parallels our duality, directly threatening PoR's requirement that physical quantities cohere across frames. It's not just a gravitational nuance—it questions relativity's ability to define consistent observables. This isn't just about angular momentum or BMS flux—it's about relativity's *raison d'être*. If PoR fails here, every relativistic prediction (e.g., gravitational waves, particle dynamics) built on LT inherits this flaw. However, paper [2]'s three puzzles pale beside those shown here and in [1]: paper [2] wrestles with covariance details while [1] and here a theory is exposed unable to define basic observables without contradiction. A researcher seeing this might not just avoid relativity with a ten-foot pole—they'd dismantle it, favoring a framework (honoring PoR) where PoR holds without such chaos.

This critique now frames relativity's collapse as systemic, not puzzle-specific, positioning paper [2] as an attempt to refine a broken theory. Their puzzle 3 remains the closest parallel, but the real issue is LT's betrayal of PoR determined in [1] and here.

References

[1] V. C. Noninski, "The Relativistic Framework Unfit for Scientific Studies," preprint, 2025, available upon request.
[2] R. Javadinezhad and M. Porrati, Phys. Rev. Lett. 132, 151604 (2024).

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