Chapter 4

Nuclear Reactions

A nuclear reaction is a process in which a nucleus reacts with another nucleus, an elementary particle, or a photon to produce, within a time of the order of 10^{-12} s or less, one or more other nuclei, and possibly other particles. The variety of reactions that have been studied is bewildering, as may be readily perceived from a consideration of the variety of bombarding particles available. They include neutrons, protons, photons, electrons, various mesons, and nuclei all the way from deuterons to uranium nuclei. The constantly advancing technology of particle accelerators (see chapter 15) has provided many of these projectiles over wide ranges of kinetic energies, for example, protons up to 500 GeV,[1] many other nuclei up to thousands of MeV per nucleon.

The phenomenon of nuclear reactions was discovered by Rutherford in 1919 when he observed that, in the bombardment of nitrogen with the 7.69-MeV α particles of RaC′ (^{214}Po), scintillations of a zinc sulfide screen persisted even when enough material to absorb all the α particles was interposed between the nitrogen and the screen. Further experiments proved the long-range particles causing the scintillations to be protons, and the results were interpreted in terms of a nuclear reaction between α particles and nitrogen to give oxygen and protons.

A. ENERGETICS

Notation. The notation used for nuclear reactions is analogous to that for chemical reactions, with the reactants on the left- and the products on the right-hand side of the equation. Thus Rutherford's first reaction may be written

$$^{14}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{17}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H}.$$

In all reactions so far observed (except those involving creation or annihilation of antinucleons) the total number of nucleons (total A) is conserved. Also conserved in nuclear reactions are charge, energy, momentum, angular momentum, statistics, and parity.

The recognition of reaction products is greatly facilitated when they are unstable, because characteristic radioactive radiations can then be obser-

[1] One GeV (gigaelectron volt) = 1000 MeV.

ved. The discovery of artificial radioactivity by Joliot and Curie thus gave enormous impetus to the field of nuclear reaction studies. The first artificially produced radionuclide observed and radiochemically characterized by them, was ^{30}P, made in the reaction

$$^{27}_{13}Al + ^{4}_{2}He \rightarrow ^{30}_{15}P + ^{1}_{0}n.$$

A short-hand notation is often used for the representation of nuclear reactions. The light bombarding particle and the light fragments (in that order) are written in parentheses between the initial and final nucleus; in this notation the two reactions mentioned above would read

$$^{14}N(\alpha, p)\,^{17}O \qquad \text{and} \qquad ^{27}Al\,(\alpha, n)\,^{30}P.$$

As indicated here, atomic numbers are commonly omitted. The symbols n, p, d, t, α, e^-, γ, π, and $\bar{p}$ are used in this notation to represent neutron, proton, deuteron, triton (^3H), alpha particle, electron, gamma ray, pi meson, and antiproton, respectively. Nuclei other than the ones mentioned are represented by their usual symbols, such as ^3He, ^{12}C, and so on, even when they are projectiles. Thus we write

$$^{139}La\,(^{12}C, 4n)\,^{147}Eu$$

for a reaction in which the bombardment of ^{139}La with ^{12}C ions results in the formation of ^{147}Eu.

Comparison of Nuclear and Chemical Reactions. Nuclear reactions, like chemical reactions, are always accompanied by a release or absorption of energy, and this is expressed by adding the term Q to the *right-hand* side of the equation. Thus a more complete statement of Rutherford's first transmutation reaction reads

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H + Q.$$

The quantity Q is called the energy of the reaction or more frequently just "the Q of the reaction." Positive Q corresponds to energy release (exoergic reaction); negative Q to energy absorption (endoergic reaction).

Here an important difference between chemical and nuclear reactions must be pointed out. In treating chemical reactions we always consider macroscopic amounts of material undergoing reactions, and consequently heats of reaction are usually given per mole or occasionally per gram of one of the reactants. In the case of nuclear reactions we usually consider single processes, and the Q values are therefore given *per nucleus transformed.* If the two are calculated on the same basis, the energy release in a representative nuclear reaction is found to be many orders of magnitude larger than that in any chemical reaction. For example, the reaction $^{14}N(\alpha, p)^{17}O$ has a Q value of -1.193 MeV or -1.912×10^{-6} erg or -4.57×10^{-14} cal *per* ^{14}N *atom* transformed. To convert 1 g atom of ^{14}N to ^{17}O would thus require an energy of $6.02 \times 10^{23} \times 4.57 \times 10^{-14}$ cal =

2.75×10^{10} cal. This is about 10^5 times as large as the largest values observed for heats of chemical reactions.

Q Values and Reaction Thresholds. The energy changes in nuclear reactions are so large that the corresponding mass changes are quite significant and observable (in contrast to the situation in chemical reactions). If the masses of all the particles participating in a nuclear reaction are known from mass-spectrographic data, as is the case for the $^{14}N(\alpha, p)^{17}O$ reaction, the Q of the reaction can be calculated. The sum of the ^{14}N and ^4He masses is 18.0056777 mass units, and the sum of the ^{17}O and ^1H masses is 18.0069585 mass units; thus an amount of energy equivalent to 0.0012808 mass unit has to be supplied to make the reaction energetically possible, or $Q = -0.0012808 \times 931.5 \text{ MeV} = -1.193 \text{ MeV}$.

Conversely, when the Q value is known experimentally (from the kinetic energies of the bombarding particle and the reaction products) it is possible to calculate the unknown mass of one of the participating nuclei from the known masses of others. By this method the masses of many radioactive nuclei have been determined (see exercise 3).

Sometimes the Q value of a reaction can be calculated even if the masses of the nuclei involved are not known, provided that the product nucleus is radioactive and decays back to the initial nucleus with known decay energy.

Consider, for example, the reaction $^{106}Pd(n, p)^{106}Rh$. The product ^{106}Rh decays with a 30-s half life and the emission of β^- particles of 3.54-MeV maximum energy to the ground state of ^{106}Pd. We can write this sequence of events as follows:

$$^{106}_{46}\text{Pd} + {}^1_0n \rightarrow {}^{106}_{45}\text{Rh} + {}^1_1\text{H} + Q;$$

$$^{106}_{45}\text{Rh} \rightarrow {}^{106}_{46}\text{Pd} + \beta^- + \bar{\nu} + 3.54 \text{ MeV}.$$

Adding the two equations, we see that the net change is just the transformation of a neutron into a proton, an electron, and an antineutrino, with accompanying energy change, or, symbolically,

$$^1_0n \rightarrow {}^1_1\text{H} + \beta^- + \bar{\nu} + Q + 3.54 \text{ MeV}.$$

Note that the symbol ^1_1H must here stand for a bare proton (evident from the charge conservation), whereas the listed "proton mass" includes the mass of one orbital electron.[2] For energy balance we therefore write

$$M_n = M_H + Q + 3.54 \text{ MeV},$$

where $M_n = 1.008665$ and $M_H = 1.007825$ mass units. Then $Q = (1.008665 - 1.007825) \times 931.5 - 3.54 = -2.76$ MeV.

[2]In general, for β^- emission and EC the masses of electrons do not have to be included in calculations when atomic masses are used. However, whenever emission of a positron is involved two electron masses have to be taken into account: one for the positron and one for the extra electron that has to leave the electron shells to preserve electrical neutrality.

In the first example calculated we found the Q value of the reaction $^{14}N(\alpha, p)^{17}O$ to be -1.19 MeV. Does that mean that this reaction can actually be produced by α particles whose kinetic energies are just over 1.19 MeV? The answer is no, for two reasons. First, in the collision between the α particle and the ^{14}N nucleus conservation of momentum requires that at least $\frac{4}{18}$ of the kinetic energy of the α particle must be retained by the products as kinetic energy. Thus only $\frac{14}{18}$ of the α particle's kinetic energy is available for the reaction. The *threshold* energy of α particles for the $^{14}N(\alpha, p)^{17}O$ reaction, that is, the kinetic energy of α particles just capable of making the reaction energetically *possible*, is $\frac{18}{14} \times 1.19 \text{ MeV} = 1.53 \text{ MeV}$. The fraction of the bombarding particle's kinetic energy that is retained as kinetic energy of the products becomes smaller with increasing mass of the target nucleus (see exercise 5).

Barriers for Charged Particles. The second reason why the α particles must have higher energies than is evident from the Q value to produce the reaction $^{14}N(\alpha, p)^{17}O$ in *good yield* is the Coulomb repulsion between the α particle and the ^{14}N nucleus. The repulsion increases with decreasing distance of separation until the α particle comes within the range of the nuclear forces of the ^{14}N nucleus. This Coulomb repulsion gives rise to the potential barrier already discussed in connection with nuclear radii in chapter 2. The height V_c of the potential barrier around a spherical nucleus of charge Z_1e and radius R_1 for a particle of positive charge Z_2e and radius R_2 may be estimated as the energy of Coulomb repulsion when the two particles are just in contact (just as in the discussion of spontaneous fission, chapter 3 section C):

$$V_c = \frac{Z_1 Z_2 e^2}{(R_1 + R_2)} \tag{4-1}$$

If R_1 and R_2 are expressed in fermis

$$V_c = 1.44 \frac{Z_1 Z_2}{R_1 + R_2} \text{ MeV}. \tag{4-2}$$

Setting $R = 1.5A^{1/3}$ fm, we get from (4-2) a value of about 3.4 MeV for the barrier height between ^{14}N and ^{4}He.[3] Classically an α particle must thus have at least $\frac{18}{14} \times 3.4 = 4.4$ MeV kinetic energy to enter a ^{14}N nucleus and produce the α, p reaction, even though the energetic threshold for the reaction is only 1.53 MeV. In the quantum-mechanical treatment of the problem there exists a finite probability for "tunneling through the barrier"

[3] We should keep in mind that the use of (4-1) and (4-2) is equivalent to assuming spherical nuclei and square-well potentials. Thus these equations serve to give only rough estimates of barrier heights, but they are quite useful for that purpose. For more sophisticated calculations of barriers we would use Woods-Saxon shapes (see chapter 2, section C, 2) for the nuclear potentials.

by lower-energy particles, but this probability drops rapidly as the energy of the particle decreases, as we saw in the discussion of α decay in chapter 3.

It follows from (4-1) that the Coulomb barrier around a given nucleus is about half as high for protons and for deuterons as it is for α particles.[4] The height of the barrier is roughly proportional to $Z^{2/3}$ (because the nuclear radius R increases approximately as $Z^{1/3}$). For the heaviest elements the potential barriers are about 12 MeV for protons and deuterons and about 25 MeV for α particles. In order to study nuclear reactions induced by charged particles, especially reactions involving heavy elements, it was therefore necessary to develop machines capable of accelerating charged particles to energies of many millions of electron volts.

In the context of nuclear reactions we emphasize again a point made already in connection with α decay, namely that Coulomb barriers affect particles not only on entering but also on leaving nuclei. For this reason a charged particle has to be excited to a rather high energy inside the nucleus before it can leak through the barrier with appreciable probability. Therefore charged particles emitted from nuclei have considerable kinetic energies (>1 MeV).

Neutrons. It is apparent that the entry of a neutron into a nucleus is not opposed by any Coulomb barrier, and even neutrons of very low energy react readily with even the heaviest nuclei. In fact, the so-called **thermal neutrons**, that is, neutrons whose energy distribution is approximately that of gas molecules in thermal equilibrium at ordinary temperatures,[5] have particularly high probabilities for reaction with target nuclei. This important effect was discovered at the University of Rome by Fermi and co-workers in 1934 in experiments on the neutron irradiation of silver; they found that the neutron-induced radioactivity was much greater when a bulk of hydrogen-containing material such as paraffin was present to modify the neutron beam. Fermi reasoned correctly that fast neutrons would lose energy in collisions with protons, that repeated collisions might reduce the energy to the thermal range, and that such slow neutrons could show large capture cross sections. Other workers found the effect to be sensitive to the temperature of the paraffin, thus demonstrating that the neutrons were actually slowed to approximately thermal energies.

[4] In estimating proton barriers of medium and heavy nuclei one usually considers the proton as a point charge ($R_2 = 0$).

[5] The energies of thermal neutrons are small fractions of an electron volt (at 20°C the most probable energy is 0.025 eV). Neutrons of somewhat higher energies (up to about 1 keV) are often called **epithermal** or **resonance neutrons**. Neutrons with kinetic energies of several thousand electron volts or more are called **fast neutrons**. The slowing down of fast neutrons is treated in chapter 6, section D.

B. CROSS SECTIONS

Definitions, Units, and Examples. We now turn to a more quantitative consideration of reaction probabilities. The probability of a nuclear process is generally expressed in terms of a cross section σ that has the dimensions of an area. This originates from the simple picture that the probability for the reaction between a nucleus and an impinging particle is proportional to the cross-sectional target area presented by the nucleus. Although this classical picture does not hold for reactions with charged particles that have to overcome Coulomb barriers or for slow neutrons (it does hold fairly well for the total probability of a fast neutron interacting with a nucleus), the cross section is a useful measure of the probability for any nuclear reaction. For a beam of particles striking a thin target, that is, a target in which the beam is attenuated only infinitesimally, the cross section for a particular process is defined by the equation

$$\mathbf{R}_i = \mathbf{I}nx\sigma_i, \tag{4-3}$$

where $\mathbf{R}_i$ is the number of processes of the type under consideration occurring in the target per unit time,
$\mathbf{I}$ is the number of incident particles per unit time,
n is the number of target nuclei per cubic centimeter of target,
σ_i is the cross section for the specified process, expressed in square centimeters, and
x is the target thickness in centimeters.

The target thickness is often given in terms of weight per unit area, which can be readily converted to nx, the number of target nuclei per square centimeter.

The total cross section for collision with a fast particle is never greater than twice[6] the geometrical cross-sectional area of the nucleus, and therefore fast-particle cross sections are rarely much larger than 10^{-24} cm^2 (radii of the heaviest nuclei are about 10^{-12} cm). Hence a cross section of 10^{-24} cm^2 is considered "as big as a barn," and 10^{-24} cm^2 has been named the **barn,** a unit generally used in expressing cross sections and often abbreviated b. The millibarn (mb, 10^{-3} b), microbarn (μb, 10^{-6} b), and nanobarn (nb, 10^{-9} b) are also commonly used.

As an example of the application of (4-3), consider a 1-h bombardment of a foil of metallic manganese, 10 mg/cm^2 thick, in a 1-μA beam of 35-MeV α particles. If the cross section of the $(\alpha, 2n)$ reaction on ^{55}Mn at this energy is 200 mb and if energy degradation of the beam in traversing the target can be neglected, how many ^{57}Co nuclei ($t_{1/2} = 270$ d) will be formed? First remember

[6]The reason why total cross sections may be as large as $2\pi R^2$ is briefly mentioned in footnote 12 on p. 31.

that $1\ \text{A} = 6.2 \times 10^{18}$ electronic charges per second so that $1\ \mu\text{A}$ of (doubly charged) α particles is 3.1×10^{12} α particles per second. The number of $(\alpha, 2n)$ reactions is, from (4-3), $3.1 \times 10^{12} \times (0.01/55) \times 6.02 \times 10^{23} \times 200 \times 10^{-27} = 6.8 \times 10^{7}$. Neglecting decay during the 1-h irradiation, we get for the number of ^{57}Co nuclei formed $3600 \times 6.8 \times 10^{7} = 2.4 \times 10^{11}$. The ^{57}Co disintegration rate at the end of the irradiation, from $dN/dt = \lambda N$, would be $[0.693/(270 \times 24 \times 60)] \times 2.4 \times 10^{11} = 4.3 \times 10^{5}\ \text{min}^{-1}$.

Equation 4-3 applies when there is a well-defined beam of particles incident on a target. Another important situation concerns a sample embedded in a uniform flux of particles incident on it from all directions. This is what happens, for example, in a nuclear reactor. It can be shown that, for a sample containing N nuclei in a flux of ϕ particles per square centimeter per second, the rate of reactions of type i, which have a cross section σ_i, is given by

$$\mathbf{R}_i = \phi N \sigma_i. \tag{4-4}$$

This applies, regardless of the shape of the sample, provided that the particle flux is not appreciably attenuated by sample absorption anywhere in the sample.

As an example, we calculate how long a 60-mg piece of Co wire has to be placed in a flux of 5×10^{13} thermal neutrons per square centimeter per second to make 1 mCi (1 millicurie $= 3.7 \times 10^{7}$ dis s^{-1}; see chapter 1, section B) of 5.27-y ^{60}Co. The cross section for the reaction ^{59}Co (n, γ) ^{60}Co is 37 b. From (4-4) we have

$$\mathbf{R} = 5 \times 10^{13} \times \frac{0.060}{59} \times 6.02 \times 10^{23} \times 37 \times 10^{-24} = 1.13 \times 10^{12}\ \text{atoms s}^{-1}.$$

From $dN/dt = \lambda N$ we find that 1 mCi of ^{60}Co corresponds to 8.87×10^{15} atoms. Thus it will take $8.87 \times 10^{15}/1.13 \times 10^{12} = 7.85 \times 10^{3}$ s, or approximately 2.2 h, to produce 1 mCi of ^{60}Co.

Beam Attenuation Measurements. If instead of a thin target we consider a thick target, that is, one in which the intensity of the incident particle beam is attenuated, the attenuation $-d\mathbf{I}$ in the infinitesimal thickness dx is given by the equation

$$-d\mathbf{I} = \mathbf{I} n \sigma_t\, dx,$$

where σ_t is the total cross section for removal of the incident particles from the beam. Integration gives

$$\mathbf{I} = \mathbf{I}_0 e^{-n\sigma_t x}. \tag{4-5}$$

Just what processes are included in σ_t depends considerably on the particular experimental arrangement, especially on the energy selectivity of the detector used to measure the transmitted beam and on the solid angle it

subtends. Thus for example, the cross section for small-angle elastic scattering may or may not be included in σ_t.

Beam attenuation measurements, of course, measure always the effect of the entire target substance, whether it is a single nuclide, an isotopic mixture, or even a compound.

Partial Cross Sections. As we emphasized in the preceding paragraph, beam attenuation or transmission experiments can be used only to determine total interaction cross sections, and (4-5) is not applicable to cross sections for specific reactions that constitute only part of the total interaction. Yet it is usually cross sections for particular processes on elementary, or even isotopically pure, substances that are of interest, such as the (n, p) reaction on ^{35}Cl or the $(\alpha, 3n)$ reaction on ^{65}Cu. Thin-target experiments are then needed so that (4-3) or (4-4) is applicable. The requirements for target thickness are particularly stringent if the cross section of interest varies rapidly with bombarding energy, as is the case for most medium-energy charged-particle reactions; the target then must be thin enough to avoid not only intensity attenuation but also appreciable energy degradation.

Sometimes the angular distribution of particles resulting from a particular process is of interest. In this case it is convenient to define a differential cross section $d\sigma/d\Omega$; this is the cross section for that part of the process in which the particles are emitted into unit solid angle at a particular angle Ω. Then the cross section for the overall process under consideration is $\sigma = \int (d\sigma/d\Omega)\, d\Omega$.

Elastic Scattering. The simplest consequence of a nuclear collision is so-called elastic scattering; this is a process that can occur at all energies and with all particles and that is not properly a reaction at all. An event is termed an elastic scattering if the particles do not change their identity during the process and if the sum of their kinetic energies (ignoring molecular and atomic excitations and bremsstrahlung) remains constant. Elastic scattering of charged particles with energies below the Coulomb barrier of the target nucleus is the Rutherford scattering described in chapter 2. As the energy of the bombarding particle is increased, the particle may penetrate the Coulomb barrier to the surface of the target nucleus, and the elastic scattering will then also have a contribution from the nuclear forces. For neutrons, of course, elastic scattering is caused by nuclear forces at all energies.

Elastic scattering may generally be considered to arise from the optical-model potential discussed in section D. With neutrons of very low energies there is also a significant contribution from so-called compound elastic scattering, since the compound nucleus formed by the amalgamation of such a neutron with the target nucleus (see section D) has a small but finite probability of emitting a neutron with all its original energy. For all other particles compound elastic scattering is negligible.

We designate the cross section for all events other than (potential) elastic scattering as the *reaction cross section.* Compound elastic scattering is formally included in the reaction cross section, although it cannot be distinguished experimentally from other elastic scattering.

Maximum Reaction Cross Sections for Neutrons. It might be expected that a nucleus that interacts with everything that hits it would have a reaction cross section of πR^2, where R is the sum of the radii of the interacting particles. As we see, this is correct at high energies only, because the wave nature of the incident particle causes the upper limit of the reaction cross section to be

$$\sigma_r = \pi (R + \lambda\!\!\!^{-})^2,$$

where $\lambda\!\!\!^{-}$ is the reduced de Broglie wavelength ($\lambda/2\pi$) of the incident particle in the center-of-mass system and may be obtained from $\lambda\!\!\!^{-} = \hbar/p$. Here p is the *relative* momentum of the two particles computed from (C-6) in appendix C.

Although cross section limits are properly derived by quantum-mechanical methods (see, e.g., B1, chapter 8), we give a semiclassical treatment that shows the essence of the problem and points up the important role played by angular-momentum considerations. We first treat reactions with incident neutrons and then proceed to discuss the additional effects of Coulomb repulsion.

Angular Momentum in Nuclear Reactions. A collision between a neutron and a target nucleus may be characterized classically by what would be the distance of closest approach of the two particles if there were no interaction between them. This distance b, usually called the **impact parameter**, is shown in figure 4-1. The angular momentum of the system is normal to the relative momentum p and of magnitude

$$L = pb. \tag{4-6}$$

The de Broglie relation between momentum and wavelength of a particle

Fig. 4-1 Collision with impact parameter b between a neutron and target nucleus with interaction radius R.

allows (4-6) to be rewritten as

$$L = \frac{\hbar b}{\lambda\!\!\!^{-}}. \tag{4-7}$$

Note that the entire treatment is in the center-of-mass system and that $\lambda\!\!\!^{-}$ is thus the reduced wavelength in that system. See appendix C for transformations between laboratory and center-of-mass systems.

As b may evidently assume any value between 0 and R, the relative angular momentum will vary continuously between 0 and $\hbar R/\lambda\!\!\!^{-}$. We know, though, that this is not acceptable; quantum mechanics requires that the component of angular momentum in a particular direction be an integer when expressed in units of $\hbar$:

$$L = l\hbar, \quad \text{where} \quad l = 0, 1, 2, \ldots. \tag{4-8}$$

Combination of (4-7) and (4-8) gives

$$b = l\lambda\!\!\!^{-}. \tag{4-9}$$

Equation 4-9 is not to be interpreted as meaning that only certain values of b are possible; such control over b would violate the uncertainty principle. Rather it means that a range of values of b corresponds to the same value of the angular momentum. In particular,

$$l\lambda\!\!\!^{-} < b < (l+1)\,\lambda\!\!\!^{-} \tag{4-10}$$

corresponds to an angular momentum of $l\hbar$. This interpretation is illustrated in figure 4-2. From this figure it can be seen that the cross-sectional area that corresponds to a collision with angular momentum $l\hbar$ is

$$\begin{aligned} \sigma_l &= \pi\lambda\!\!\!^{-2}[(l+1)^2 - l^2] \\ &= \pi\lambda\!\!\!^{-2}(2l+1). \end{aligned} \tag{4-11}$$

If it is assumed that each particle hitting the nucleus causes a reaction, then

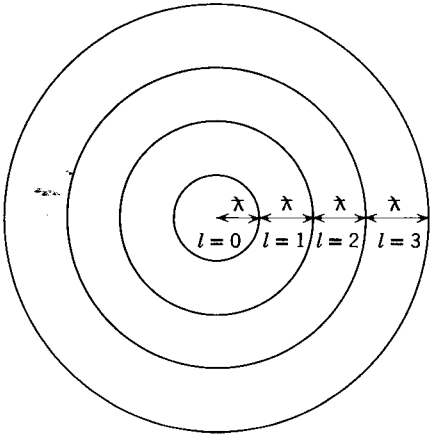

Fig. 4-2 The incident beam is perpendicular to the plane of the figure. The particles with a particular l are considered to strike within the designated ring.

(4-11) gives the partial cross section for a nuclear reaction characterized by angular momentum $l\hbar$, and the reaction cross section may be obtained by summing (4-11) over all values of l from 0 to the maximum l_m:

$$\sigma_r = \pi\lambda\!\!\!^{-2} \sum_0^{l_m} (2l + 1). \tag{4-12}$$

The summation in (4-12) may be easily evaluated if it is recalled that the sum of the first N integers is equal to $[N(N + 1)]/2$. The expression for the reaction cross section becomes

$$\sigma_r = \pi\lambda\!\!\!^{-2} (l_m + 1)^2. \tag{4-13}$$

The maximum value of l may be estimated from (4-9) by limiting the maximum impact parameter to the interaction radius R:

$$l_m = \frac{R}{\lambda\!\!\!^{-}}. \tag{4-14}$$

Substitution of (4-14) into (4-13) yields the result already given on p. 118 for the maximum possible reaction cross section:

$$\sigma_r = \pi (R + \lambda\!\!\!^{-})^2. \tag{4-15}$$

This result suggests the possibility of nuclear-reaction cross sections that are several orders of magnitude larger than the geometrical cross section of the nucleus, a possibility that is realized in slow-neutron reactions. The largest thermal-neutron cross section known is that of ^{135}Xe, 2.65×10^6 b. (See also section E.)

In the quantum-mechanical treatment of the problem (B1) the result for the total reaction cross section is not (4-12), but

$$\sigma_r = \pi\lambda\!\!\!^{-2} \sum_{l=0}^{\infty} (2l + 1) T_l, \tag{4-16}$$

where T_l is defined as the **transmission coefficient** for the reaction of a neutron with angular momentum l and may have values between zero and one; it represents the fraction of incident particles with angular momentum l that penetrate within the range of nuclear forces. Our semiclassical treatment assigns unity to T_l for all values of l up to and including l_m, as defined in (4-14); for all higher values of l, the transmission coefficients are zero. The role of angular momentum here is analogous to the one that it plays in β and γ emission, discussed in chapter 3. It should be mentioned here that the semiclassical result is quite right for $R/\lambda\!\!\!^{-} < 1$, where the only contribution comes from $l = 0$ and the reaction cross section has $\pi\lambda\!\!\!^{-2}$ as its upper limit.

Centrifugal Barrier. Expression 4-14 for the maximum l value can be reinterpreted to mean that a particle that approaches a nucleus with relative angular momentum l must have a reduced de Broglie wavelength

$\lambda\!\!\!^{-} \leq R/l$. Since $\epsilon = p^2/2\mu = \hbar^2/2\mu\lambda\!\!\!^{-2}$, where ϵ is the relative kinetic energy, p the relative momentum, and μ the reduced mass of the system, we have the condition

$$\epsilon \geq \frac{l^2\hbar^2}{2\mu R^2}, \tag{4-17}$$

where R may be taken as the sum of the radii of projectile particle and target nucleus. Condition 4-17 implies that, quite apart from any Coulomb barrier, there is for particles of angular momentum l an additional barrier, called the centrifugal barrier. In the quantum-mechanical treatment of the problem l^2 is replaced by $l(l+1)$, so that the proper expression for the centrifugal barrier or centrifugal potential is

$$V_l = \frac{l(l+1)\hbar^2}{2\mu R^2}. \tag{4-18}$$

Reaction Cross Sections with Charged Particles. The effect of the Coulomb repulsion on a reaction cross section may be easily estimated within the spirit of the semiclassical analysis. The Coulomb repulsion will bring the relative kinetic energy of the system from ϵ when the particles are very far apart to $\epsilon - V_c$ when the two particles are just touching, where V_c is the Coulomb barrier:

$$V_c = \frac{Z_a Z_A e^2}{R}, \tag{4-19}$$

and where Z_a and Z_A are the atomic numbers of incident particle and target nucleus, respectively. Further, the deflection of the particles causes the maximum impact parameter that leads to a reaction to be less than R, as illustrated in figure 4-3. From this figure it is seen that the trajectory of the particle is tangential to the nuclear surface when it approaches with the maximum impact parameter b_m and that the relative momentum at the

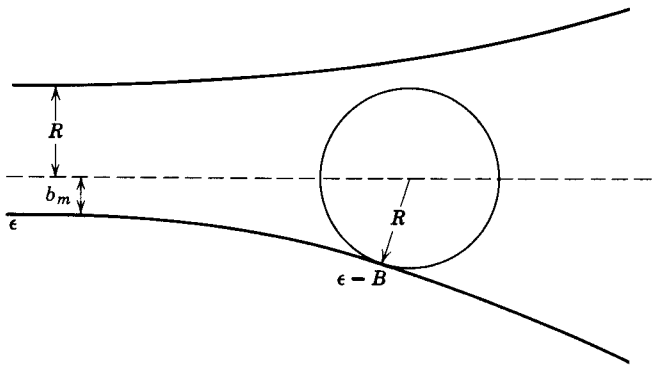

Fig. 4-3 Classical trajectories for charged particles with impact parameters R and b_m.

point of contact is

$$p = (2\mu)^{1/2}(\epsilon + V_c)^{1/2} = (2\mu\epsilon)^{1/2}\left(1 - \frac{V_c}{\epsilon}\right)^{1/2}, \tag{4-20}$$

where μ is the reduced mass of the system. The magnitude of the maximum angular momentum is then obtained from the product of the interaction radius and the relative momentum:

$$L_m = R\,(2\mu\epsilon)^{1/2}\left(1 - \frac{V_c}{\epsilon}\right)^{1/2}. \tag{4-21}$$

Recognizing that $(2\mu\epsilon)^{1/2}$ is the relative momentum of the two particles when they are far apart, we obtain from (4-21) in conjunction with (4-6)

$$b_m = R\left(1 - \frac{V_c}{\epsilon}\right)^{1/2} \tag{4-22}$$

for the maximum impact parameter. Equation 4-22 has meaning only for $\epsilon \geq V_c$; for lower ϵ the Coulomb potential, classically, prevents nuclear reactions. Thus the Coulomb barrier diminishes the l_m of (4-14) by a factor of $(1 - V_c/\epsilon)^{1/2}$. The upper limit for the capture of charged particles can be estimated as the area of the disk of radius b_m:

$$\sigma_r = \pi R^2\left(1 - \frac{V_c}{\epsilon}\right). \tag{4-23}$$

It is to be noted that the method of estimating the upper limit to the reaction cross section for charged particles is different from that for neutrons, where the value of l_m was found and substituted directly into (4-13). This procedure would not be appropriate for charged particles, as the Coulomb barrier has an important effect on the transmission coefficients of (4-16). In particular, it may be seen from (4-21) that $l_m \to 0$ as $\epsilon \to V_c$ for charged particles and from (4-14) that $l_m \to 0$ as $\epsilon \to 0$ for neutrons. However, the Coulomb barrier causes the transmission coefficient for charged particles to approach zero under these circumstances, whereas that for the neutron remains finite. The result is a vanishing cross section for charged particles of energies approaching that of the Coulomb barrier to be compared with the upper limit of $\pi(R + \lambda\!\!\!^{-})^2$ given in (4-15) for neutrons of very low energy. Further, since the Coulomb barrier is the most important factor in determining reaction cross sections with charged particles, (4-23) is an estimate of the reaction cross section rather than just its upper limit.

Equation 4-23, though approximate, has been useful for the estimation of reaction cross sections for charged particles, particularly when the Coulomb barrier is changed to an "effective" Coulomb barrier (see, e.g., D1) to allow for tunneling through the diffuse nuclear surface. Again, the complete analysis of the reaction cross section is properly carried out with (4-16), and the effect of the Coulomb interaction appears in the transmission coefficients (S1, H1).

C. TYPES OF EXPERIMENTS

Nuclear reactions are studied in a variety of ways. Among the important types of experimental information we usually wish to obtain are the reaction cross section, in particular its variation with incident energy, and the energy spectra and angular distributions of the reaction products. The types of experiments performed to obtain these data and the kind of information deduced from them are sketched in the following paragraphs.[7]

Excitation Functions. Frequently the variation of a particular reaction cross section with incident energy is of interest; the relation between the two is called an excitation function. Examples of excitation functions are shown in figures 4-4 and 4-5. Variable-energy beams are obtainable from various types of accelerators (see chapter 15). If only a fixed energy source is available, energy degradation by absorption is resorted to, with resulting spread in beam energy (see chapter 6). Methods for determining beam energies and beam intensities, both essential for accurate, absolute excitation function measurements, are discussed in chapter 15, section D. The determination of an absolute cross section further requires measurement of the number of reactions in the target, usually via determination of the

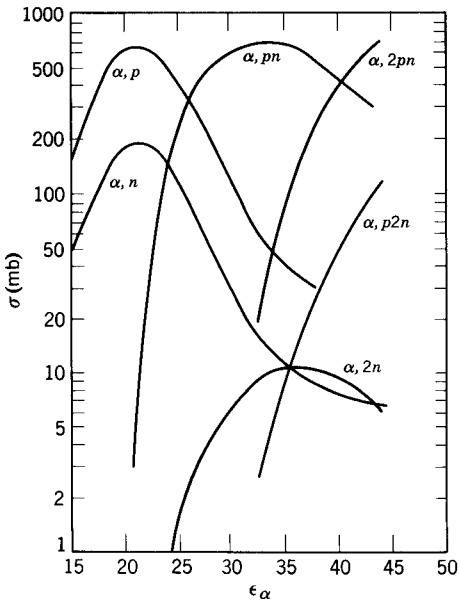

Fig. 4-4 Excitation functions for various reactions between α particles and ^{54}Fe nuclei. The abscissa is the kinetic energy of the α particle in the laboratory system. [Data from F. S. Houck and J. M. Miller, *Phys. Rev.* **123**, 231 (1961).]

[7] An important class of experiments, which is not discussed here, is aimed primarily at obtaining information on the energy states of product nuclei rather than on the mechanisms of the reactions. This field (reaction spectroscopy) is considered in chapter 8, section F.

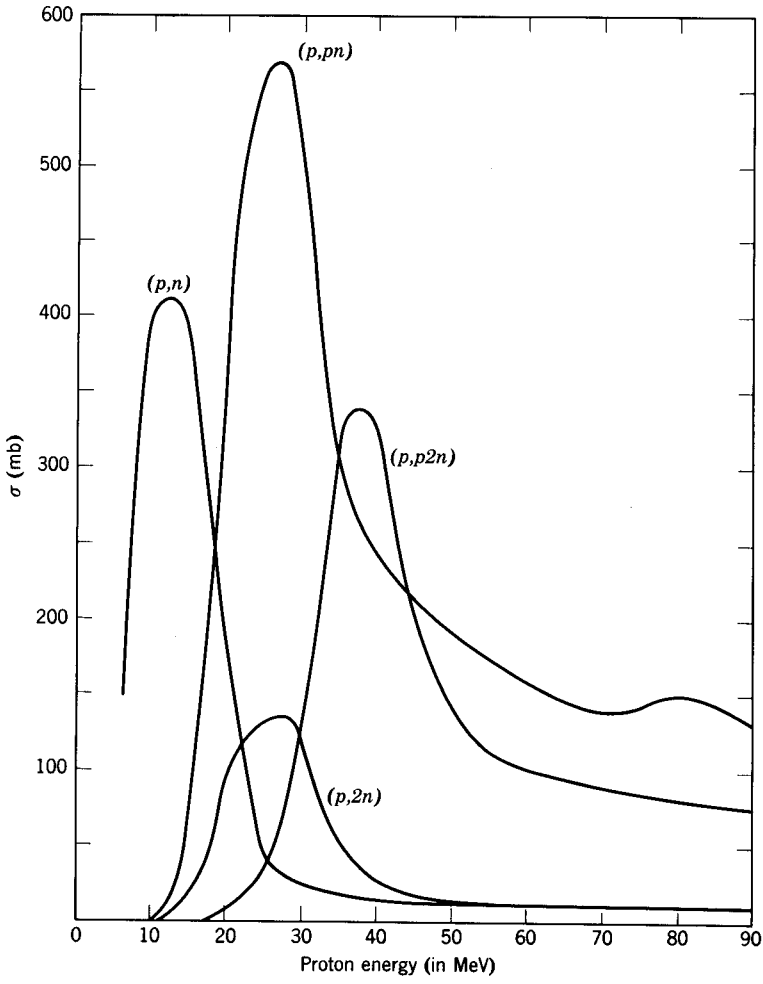

Fig. 4-5 Excitation functions for proton-induced reactions on ^{63}Cu. [From J. W. Meadows, *Phys. Rev.* **91**, 885 (1953).]

absolute disintegration rate of a radioactive product; the relevant techniques are discussed in chapter 8, section G.

The *shape* of an excitation function can often be determined in a much simpler way by the so-called stacked-foil method, that is, by exposing several target foils in the same beam, with appropriate energy-degrading foils interposed. An absolute calibration, if desired, can then be done at a single energy. In general, the higher the energy of a bombarding particle, the more complex the possible reactions. For example, with a few exceptions among the lightest nuclides, thermal neutrons can induce (n, γ) reactions only. With neutrons of several million electron volts kinetic

energy, (n, p) reactions become possible and prevalent; at still higher energies $(n, 2n)$, (n, α), and (n, np) reactions set in. In the bombarding energy range up to about 50 MeV the cross section of a given reaction rises with increasing bombarding energy from threshold to some maximum value that is usually reached about 10 MeV above threshold and then drops again to some low value; the drop is accompanied by the rise of cross sections for other more complex reactions. This behavior is illustrated by the excitation functions shown in figure 4-5.

Excitation functions provide some information about the probabilities for the emission of various kinds of particles and combinations of particles in nuclear reactions because the formation of a given product implies what particles were ejected from the target nuclide. For example, figure 4-4 shows that in the reactions of α particles with ^{54}Fe the emission of a single proton is about three times as likely as that of a single neutron and the emission of a proton-neutron pair is about 50 times more probable than that of two neutrons. It is not possible from these data alone, though, to know whether the proton-neutron pair was emitted as a deuteron; in this instance it very likely was not.

It is possible to get some information about the kinetic energies of the emitted particles both from the energies at which the various excitation functions reach their maxima and from the slopes of the excitation functions, but these are at best rather crude estimates. Excitation functions will not yield any information about the angular distribution of the emitted particles.

Total Reaction Cross Sections. These quantities (which we designate as σ_r) are not as easily determined as individual activation cross sections. Summing of all experimentally measured excitation functions for individual reactions rarely yields an excitation function for σ_r since, for most target-projectile combinations, some reactions lead to stable products and thus cannot be measured by the activation technique. For example, in the $^{54}\text{Fe} + \alpha$ system illustrated in figure 4-4, the (α, γ) reaction leads to stable ^{58}Ni, the $(\alpha, 2p)$ reaction to stable ^{56}Fe, and so on. However, such unmeasured cross sections can often be estimated quite well by statistical theory (see section D) and, if the unmeasured contributions are not too large, this combination of experimental and calculated partial cross sections can lead to good data on excitation functions for σ_r.

The other general method for σ_r is to measure the attenuation of a beam, that is, to determine directly the quantity $(I - I_0)/I_0$ [see (4-5)]. In energy regions in which σ_r varies rapidly with projectile energy, this method is difficult to apply because, on the one hand, the target must be kept thin enough to minimize energy degradation of the beam, but on the other hand, a thin target also produces a small attenuation in intensity, which is hard to measure with good accuracy.

Particle Spectra. In contrast to excitation functions the second type of

experiment focuses attention on the energy and angular distributions of the emitted particles. In its simplest form this information may be collected experimentally by detection of the emitted particles in an energy-sensitive detector placed at various angles θ with respect to the incident beam. The quantity that is usually reported is $\partial^2\sigma/\partial\epsilon\partial\Omega$, which is a function of the kinetic energy ϵ of the emitted particle, and of the angle of emission θ. This quantity is the differential cross section for the emission of the particle with kinetic energy between ϵ and $\epsilon + d\epsilon$ into an element of solid angle $d\Omega$ at an angle θ with respect to the incident beam. In the laboratory system of coordinates the solid angle $d\Omega$ may be roughly approximated by dividing the area of the detector normal to the emission direction of the particle by the square of the distance between the target and the detector. Bearing in mind that the differential cross section is a function of ϵ and θ, the total cross section for the emission of the particle is obtained by integrating over all angles and energies:

$$\sigma = 2\pi \int_0^\infty \int_0^\pi \frac{\partial^2\sigma}{\partial\epsilon\,\partial\Omega} \sin\theta\, d\theta\, d\epsilon.$$

Some examples of energy and angular distributions are shown in figures 4-6 and 4-7.

An obvious limitation on the information that measurements of particle spectra provide for theoretical analysis lies in the lack of knowledge about the other particles that may be emitted in the same event with the one being detected. This difficulty may be circumvented either by using an energy so low that the probability for the emission of more than one

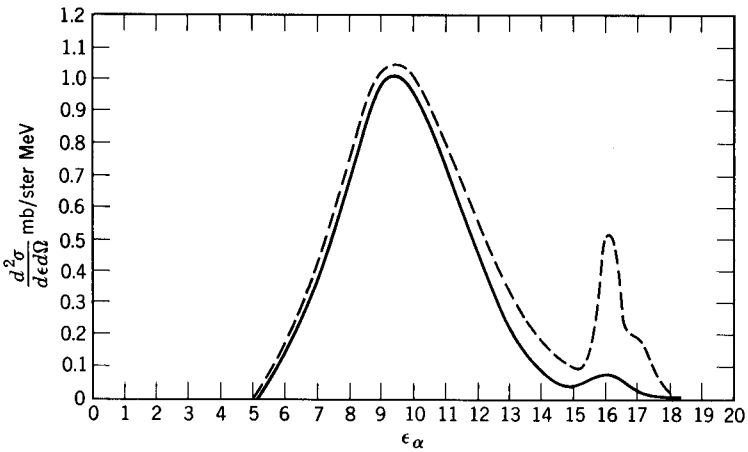

Fig. 4-6 Spectrum of α particles from Ni(p, α)Co for a nickel target of normal isotopic composition bombarded with 17.6-MeV protons. The dashed curve is the spectrum at 30° with respect to the incident beam, the solid curve is that at 120°. [Reproduced from R. Sherr and F. P. Brady, *Phys. Rev.* **124**, 1928 (1961).]

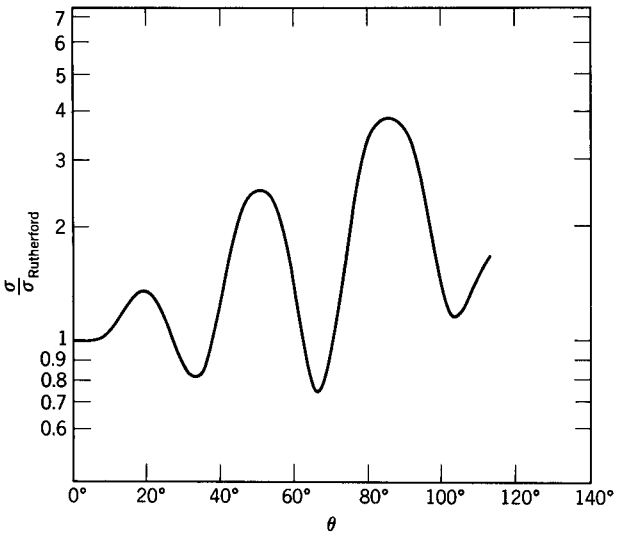

Fig. 4-7 Angular distribution of 22-MeV protons elastically scattered by nickel. The ordinate is the observed scattering cross section divided by the scattering expected from purely Coulombic interaction (Rutherford scattering). The abscissa is the scattering angle in the center-of-mass system. [Data from B. L. Cohen and R. V. Neidigh, *Phys. Rev.* **93**, 282 (1954).]

particle is negligible or by having several detectors and demanding coincidences among them before an event is recorded. The latter technique is fine in principle, but the rate of gathering information with it diminishes rapidly as the multiplicity of coincidence requirements increases. This limitation becomes less severe with the use of multiple-detector arrays.

In addition to measuring the energy and direction of emission of a particle, it is often necessary to establish its identity, that is, to determine whether it is a proton, deuteron, α particle, pion, carbon nucleus, or whatever, and to distinguish it from other particles that may be emitted from the same target. This is generally accomplished through measurement of some appropriate combination of quantities such as total kinetic energy E, specific energy loss dE/dx (cf. chapter 6), momentum p, velocity v, and mass m. The instruments and techniques used for such measurements are discussed in chapters 7 and 8.

Radiochemical Recoil Measurements. The techniques mentioned in the preceding paragraph allow unambiguous identification by A and Z for light particles (perhaps up to $A \approx 25$) emitted in nuclear reactions. For heavier fragments and product nuclei one has to resort to other methods to obtain angular distributions and kinetic-energy spectra. Specifically one can combine the activation technique with angular and energy measurements provided the product of interest is radioactive.

The experimental techniques that are employed for the measurements vary in complexity. In the simplest experiment, catcher foils placed before and behind the target determine the fraction of a given product that recoils out of the target in the forward and in the backward directions. This measurement is sensitive to the relative amounts of momentum carried away by the particles emitted in the forward and in the backward direction. In the more elegant experiment stacks of very thin (thin compared to the range of the recoil product) catcher foils are placed at various angles with respect to the incident beam, and a direct measurement of the angular and kinetic-energy distribution of the products is made. For this type of experiment the target itself must, of course, also be thin relative to the recoil ranges of interest. Reaction studies by recoil techniques are reviewed in A1.

D. REACTION MODELS AND MECHANISMS

Before proceeding to a phenomenological survey of the various types of nuclear reactions in the following sections, we give in this section a brief account of the theoretical framework in which nuclear reactions are discussed. When we remember that, even for the interaction between two individual nucleons, a complete theory in terms of nuclear forces is still lacking, it should not come as a surprise that the subject of nuclear reactions is far too complex and multifaceted to be understood in terms of a single, exact theory. Instead, it has been necessary to rely on simplified models for the description, systematization, and "understanding" of the observed phenomena as well as for predictive purposes. These models have indeed proved very useful.

1. *Optical Model*

The earliest model considered in attempts to understand cross sections for nuclear processes was one in which the interactions of the incident particle with the nucleons of the nucleus were replaced by its interaction with a potential-energy well. Although abandoned in the 1930s when it could not account for the phenomenon of slow-neutron resonances (see below), the model was revived in modified form in 1949 (F1) for the description of nuclear reactions at higher energies ($\approx$100 MeV), and since that time it has been used fruitfully in the interpretation of elastic-scattering and total-reaction cross sections at energies down to a few million electron volts.

The analogy with a beam of light passing through a transparent glass ball has caused the model to be called the optical model. In its simplest form it represents the nucleus by a square-well potential V_0 MeV deep and R fm wide as illustrated in figure 4-8. The kinetic energy of a neutron entering