Two Inconsistent Field Expressions for One System, In One Frame

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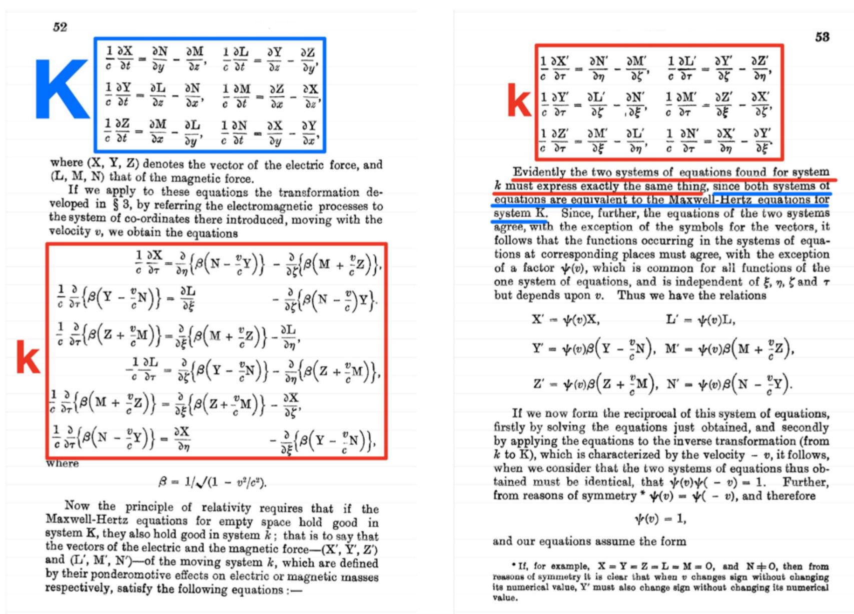
In a foundational electromagnetic analysis, a single system in a stationary frame K yields in a moving frame k field expressions $X = X'$ and $Y = \frac{Y'}{\beta} + \frac{v}{c}N$, because "[e]vidently the two systems of equations found for system k must express exactly the same thing", with $v \neq 0$, $\beta \neq 1$ (cf. Figure 1, showing also the meaning of the parameters). These expressions tie to K's system, $\frac{1}{c}\frac{\partial X}{\partial t} = \cdots$, $\frac{1}{c}\frac{\partial Y}{\partial t} = \cdots$, presumed consistent with k's $\frac{1}{c}\frac{\partial X'}{\partial \tau} = \cdots$, $\frac{1}{c}\frac{\partial Y'}{\partial \tau} = \cdots$. Yet, rewriting K's equations with k's terms distorts them: $\frac{1}{c}\frac{\partial X}{\partial t} \to \frac{1}{c}\frac{\partial X'}{\partial \tau} = \ldots$, while $\frac{1}{c}\frac{\partial Y}{\partial t} \to \frac{1}{c}\frac{\partial}{\partial \tau}\left(\frac{Y'}{\beta} + \frac{v}{c}N\right) = \ldots$. Unless $v = 0, \beta = 1$, which violates the initial conditions, this skews K's form—$Y \neq Y'$, and the derivative misaligns with k's structure. The text asserts both expressions, $\frac{1}{c}\frac{\partial Y'}{\partial \tau} = \cdots$ and $\frac{1}{c}\frac{\partial}{\partial \tau}\left(\frac{Y'}{\beta} + \frac{v}{c}N\right) = \cdots$, hold in k without resolution; for $v \neq 0$, they imply incompatible field behaviors under identical conditions, an unaddressed duality. This persists notwithstanding that $Y = \frac{Y'}{\beta} + \frac{v}{c}N$ is dimensionally inconsistent in the SI unit system, a flaw cloaked by Gaussian units or ad hoc adjustments in natural units, masking the flaw in unit-agnostic derivations.



52

$$\frac{1}{c}\frac{\partial X}{\partial t} = \frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}, \quad \frac{1}{c}\frac{\partial L}{\partial t} = \frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y},$$
$$\frac{1}{c}\frac{\partial Y}{\partial t} = \frac{\partial L}{\partial z} - \frac{\partial N}{\partial x}, \quad \frac{1}{c}\frac{\partial M}{\partial t} = \frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z},$$
$$\frac{1}{c}\frac{\partial Z}{\partial t} = \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}, \quad \frac{1}{c}\frac{\partial N}{\partial t} = \frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x},$$

where (X, Y, Z) denotes the vector of the electric force, and (L, M, N) that of the magnetic force.

If we apply to these equations the transformation developed in § 3, by referring the electromagnetic processes to the system of co-ordinates there introduced, moving with the velocity v, we obtain the equations

$$\frac{1}{c}\frac{\partial X}{\partial \tau} = \frac{\partial}{\partial \eta}\left\{\beta\left(N - \frac{v}{c}Y\right)\right\} - \frac{\partial}{\partial \zeta}\left\{\beta\left(M + \frac{v}{c}Z\right)\right\},$$
$$\frac{1}{c}\frac{\partial}{\partial \tau}\left\{\beta\left(Y - \frac{v}{c}N\right)\right\} = \frac{\partial L}{\partial \xi} - \frac{\partial}{\partial \zeta}\left\{\beta\left(N - \frac{v}{c}\right)Y\right\},$$
$$\frac{1}{c}\frac{\partial}{\partial \tau}\left\{\beta\left(Z + \frac{v}{c}M\right)\right\} = \frac{\partial}{\partial \xi}\left\{\beta\left(M + \frac{v}{c}Z\right)\right\} - \frac{\partial L}{\partial \eta},$$
$$\frac{1}{c}\frac{\partial L}{\partial \tau} = \frac{\partial}{\partial \zeta}\left\{\beta\left(Y - \frac{v}{c}N\right)\right\} - \frac{\partial}{\partial \eta}\left\{\beta\left(Z + \frac{v}{c}M\right)\right\},$$
$$\frac{1}{c}\frac{\partial}{\partial \tau}\left\{\beta\left(M + \frac{v}{c}Z\right)\right\} = \frac{\partial}{\partial \xi}\left\{\beta\left(Z + \frac{v}{c}M\right)\right\} - \frac{\partial X}{\partial \zeta},$$
$$\frac{1}{c}\frac{\partial}{\partial \tau}\left\{\beta\left(N - \frac{v}{c}Y\right)\right\} = \frac{\partial X}{\partial \eta} - \frac{\partial}{\partial \xi}\left\{\beta\left(Y - \frac{v}{c}N\right)\right\},$$

where

$$\beta = 1/\sqrt{(1 - v^2/c^2)}.$$

Now the principle of relativity requires that if the Maxwell-Hertz equations for empty space hold good in system K, they also hold good in system k; that is to say that the vectors of the electric and the magnetic force—(X′, Y′, Z′) and (L′, M′, N′)—of the moving system k, which are defined by their ponderomotive effects on electric or magnetic masses respectively, satisfy the following equations :—

53

$$\frac{1}{c}\frac{\partial X'}{\partial \tau} = \frac{\partial N'}{\partial \eta} - \frac{\partial M'}{\partial \zeta}, \quad \frac{1}{c}\frac{\partial L'}{\partial \tau} = \frac{\partial Y'}{\partial \zeta} - \frac{\partial Z'}{\partial \eta},$$
$$\frac{1}{c}\frac{\partial Y'}{\partial \tau} = \frac{\partial L'}{\partial \zeta} - \frac{\partial N'}{\partial \xi}, \quad \frac{1}{c}\frac{\partial M'}{\partial \tau} = \frac{\partial Z'}{\partial \xi} - \frac{\partial X'}{\partial \zeta},$$
$$\frac{1}{c}\frac{\partial Z'}{\partial \tau} = \frac{\partial M'}{\partial \xi} - \frac{\partial L'}{\partial \eta}, \quad \frac{1}{c}\frac{\partial N'}{\partial \tau} = \frac{\partial X'}{\partial \eta} - \frac{\partial Y'}{\partial \xi}.$$

Evidently the two systems of equations found for system k must express exactly the same thing, since both systems of equations are equivalent to the Maxwell-Hertz equations for system K. Since, further, the equations of the two systems agree, with the exception of the symbols for the vectors, it follows that the functions occurring in the systems of equations at corresponding places must agree, with the exception of a factor $\psi(v)$, which is common for all functions of the one system of equations, and is independent of ξ, η, ζ and τ but depends upon v. Thus we have the relations

$$X' = \psi(v)X, \quad L' = \psi(v)L,$$
$$Y' = \psi(v)\beta\left(Y - \frac{v}{c}N\right), \quad M' = \psi(v)\beta\left(M + \frac{v}{c}Z\right),$$
$$Z' = \psi(v)\beta\left(Z + \frac{v}{c}M\right), \quad N' = \psi(v)\beta\left(N - \frac{v}{c}Y\right).$$

If we now form the reciprocal of this system of equations, firstly by solving the equations just obtained, and secondly by applying the equations to the inverse transformation (from k to K), which is characterized by the velocity $-v$, it follows, when we consider that the two systems of equations thus obtained must be identical, that $\psi(v)\psi(-v) = 1$. Further, from reasons of symmetry * $\psi(v) = \psi(-v)$, and therefore

$$\psi(v) = 1,$$

and our equations assume the form

* If, for example, $X = Y = Z = L = M = 0$, and $N \neq 0$, then from reasons of symmetry it is clear that when v changes sign without changing its numerical value, Y′ must also change sign without changing its numerical value.

Figure 1. Excerpt from a historical derivation, showing dual field expressions in the moving frame k.